



# Complex Network Clustering by Multiobjective Discrete Particle Swarm Optimization Based on Decomposition

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# 主要内容

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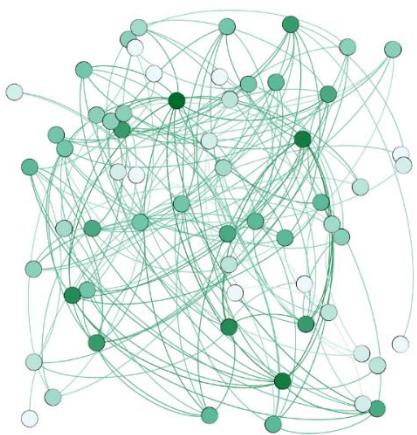




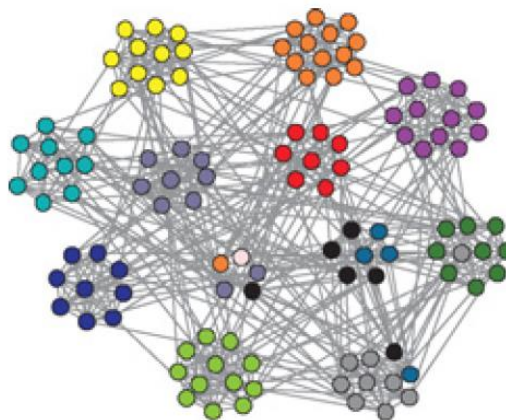
# 研究背景

## ➤ 复杂网络

- 小世界
- 集群即集聚程度 (clustering coefficient)
- 幂律 (power law)



海豚社交网络



足球网络





# 研究背景

## ➤ 网络定义

### 1. 非符号网络 (unsigned network) $G=(V,E)$

强感知  $\forall i \in S, k_i^{in} > k_i^{out}$

弱感知  $\sum_{i \in S} k_i^{in} > \sum_{i \in S} k_i^{out}$

$$k_i^{in} = \sum_{i,j \in S} A_{ij}$$

$$k_i^{out} = \sum_{i,j \notin S} A_{ij}$$

### 2. 符号网络 (signed network) $G=(V,PE,NE)$

强感知 社区内，积极连接大于消极连接

$$\forall i \in S, (k_i^+)^{in} > (k_i^-)^{in}$$

弱感知 社区内，积极连接密集；社区间消极连接密集

$$\begin{cases} \sum_{i \in S} (k_i^+)^{in} > \sum_{i \in S} (k_i^+)^{out} \\ \sum_{i \in S} (k_i^-)^{out} > \sum_{i \in S} (k_i^-)^{in} \end{cases}$$





# 问题描述

## ➤ 网络聚类

**Girvan & Newman** 提出模块度函数 (**Q函数**)

$$Q = \frac{1}{2m} \sum_{i,j} (A_{ij} - k_i k_j / 2m) \delta(i, j)$$

**G'omez** 提出 (**SQ函数**)

$$SQ = \frac{1}{2\omega^+ + 2\omega^-} \sum_{i,j} \left( \omega_{ij} - \left( \frac{\omega_i^+ \omega_j^+}{2\omega^+} - \frac{\omega_i^- \omega_j^-}{2\omega^-} \right) \right) \delta(i, j)$$

存在局限性





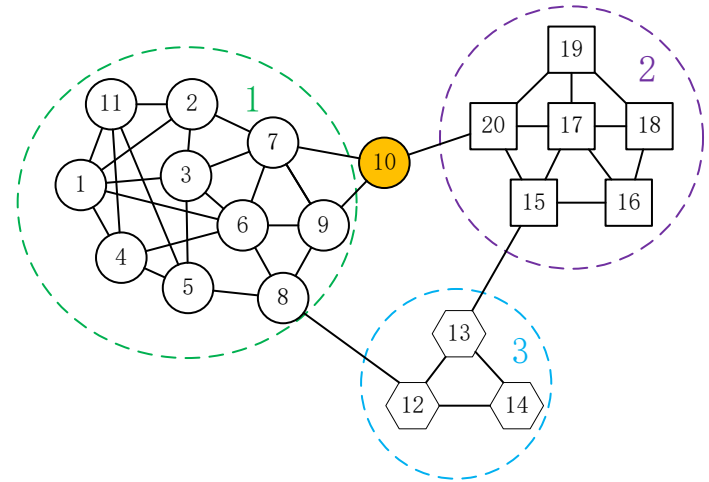
# 问题描述

## ➤ 模块密度 (modularity density)

$$S = (V_1, V_2, \dots, V_k)$$

$$D = \sum_{i=1}^k \frac{\boxed{L(V_i, V_i)} - \boxed{L(V_i, \bar{V}_i)}}{|V_i|}$$

maximize minimize



$$L(V_1, \bar{V}_2) = \sum_{i \in V_1, j \in \bar{V}_2} A_{ij} \quad L(V_1, V_2) = \sum_{i \in V_1, j \in V_2} A_{ij}$$

M. Gong, L. Ma, Q. Zhang, and L. Jiao, "Community detection in networks by using multiobjective evolutionary algorithm with decomposition," *Phys. A Stat. Mech. its Appl.*, vol. 391, no. 15, pp. 4050–4060, 2012.





# 问题描述

## ➤ 目标函数（非符号网络）

negative ratio association (NRA)  kernel k-means (KKM)  
ratio cut (RC)

$$\min = \begin{cases} \boxed{NRA = - \sum_{i=1}^k \frac{L(V_i, V_i)}{|V_i|}} \\ \boxed{RC = \sum_{i=1}^k \frac{L(V_i, \bar{V}_i)}{|V_i|}} \end{cases} \quad \text{➔} \quad \boxed{KKM = 2(n - k) - \sum_{i=1}^k \frac{L(V_i, V_i)}{|V_i|}}$$





# 问题描述

## ➤ 目标函数（符号网络）

$$\min = \begin{cases} SRA = - \sum_{i=1}^k \frac{L^+(V_i, V_i) - L^-(V_i, V_i)}{|V_i|} \\ RC = \sum_{i=1}^k \frac{L^+(V_i, \bar{V}_i) - L^-(V_i, \bar{V}_i)}{|V_i|} \end{cases}$$

$$L^+(V_i, V_j) = \sum_{i \in V_i, j \in V_j} A_{ij} \quad A_{ij} > 0$$

$$L^-(V_i, V_j) = \sum_{i \in V_i, j \in V_j} |A_{ij}| \quad A_{ij} < 0$$



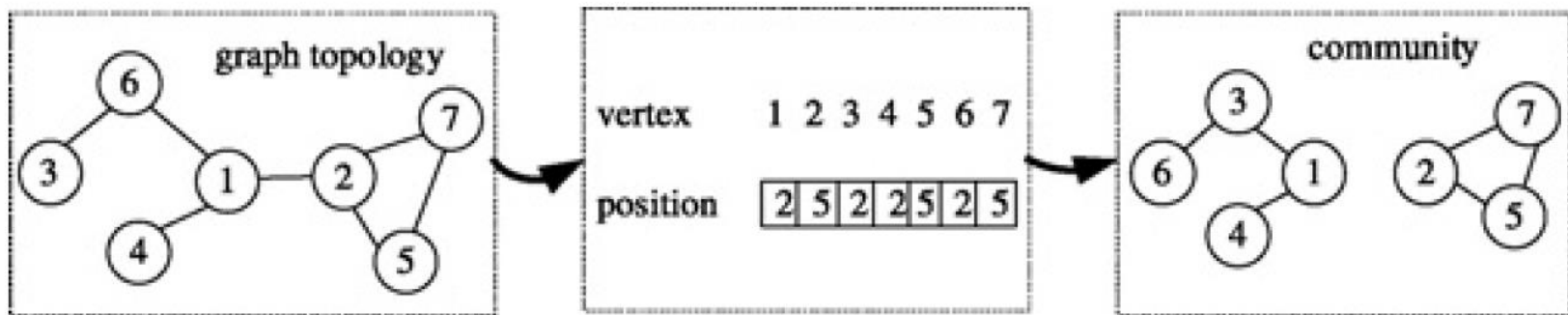




# 算法框架

## ➤ 粒子状态定义

### 位置定义



### 速度定义

$$V_i = \{v_1, v_2, \dots, v_n\}$$

如果  $v_i = 1$       粒子飞行

如果  $v_i = 0$       粒子不动





# 算法框架

## ➤ 粒子状态更新

$$V_i = \mathit{sig}(\omega V_i + c_1 r_1 (X_i \oplus Pbest_i) + c_2 r_2 (X_i \oplus Gbest))$$

$$\begin{cases} y_i = 1 & \text{if } \mathit{rand}(0,1) < \mathit{sigmoid}(x_i) \\ y_i = 0 & \text{if } \mathit{rand}(0,1) \geq \mathit{sigmoid}(x_i) \end{cases}$$

例:

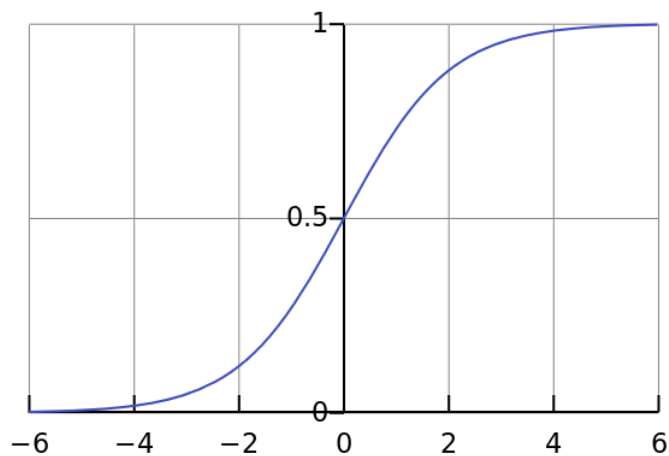
$$X_i = \{1, \mathbf{2}, \mathbf{2}, \mathbf{4}, 2, 2, \mathbf{7}\}$$

$$Pbest = \{1, \mathbf{1}, \mathbf{1}, \mathbf{1}, 2, 2, \mathbf{2}\}$$

$$V1 = \{0, \mathbf{1}, \mathbf{1}, \mathbf{1}, 0, 0, \mathbf{1}\} \quad r1c1 = 1.4$$

$$V2 = \{1, 0, 0, 1, 0, 1, 0\} \quad r2c2 = 0.7$$

$$\mathit{sig}(X) = \{0, 1, 1, 1, 0, 0, 1\}$$



$$\mathit{sigmoid}(x) = \frac{1}{1 + e^{-x}}$$





# 算法框架

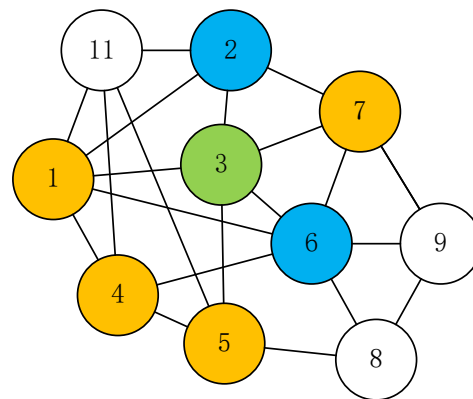
## ➤ 粒子状态更新

$$x_i^t = x_i^t \otimes v_i^t$$

定义 $\otimes$ 操作，若 $v=1$ ， $x$ 更新为其邻接节点所在最多的社区

$$\begin{cases} x_{2i} = x_{1i} & \text{if } v_i = 0 \\ x_{2i} = Nbest_i & \text{if } v_i = 1 \end{cases}$$

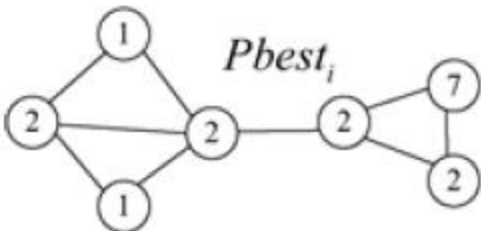
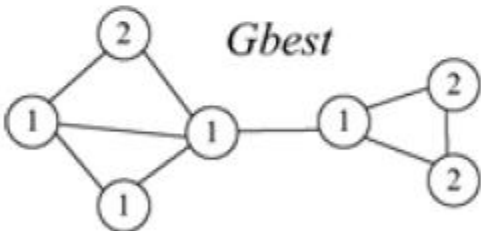
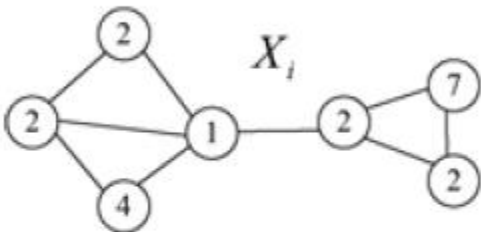
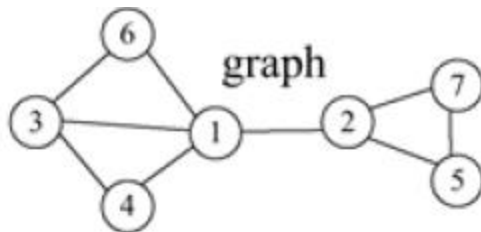
$$Nbest_i = \operatorname{argmax}_r \sum_{j \in N} \varphi(x_{1j}, r)$$





# 算法框架

## ➤ 粒子状态更新



$$X_i = [x_1, x_2, x_3, x_4, x_5, x_6, x_7]$$

$$= [1 \ 2 \ 2 \ 4 \ 2 \ 2 \ 7]$$

$$G_{best} = [g_1, g_2, g_3, g_4, g_5, g_6, g_7]$$

$$= [1 \ 1 \ 1 \ 1 \ 2 \ 2 \ 2]$$

$$P_{best}_i = [p_1, p_2, p_3, p_4, p_5, p_6, p_7]$$

$$= [2 \ 2 \ 2 \ 1 \ 2 \ 1 \ 7]$$

$$G_{best} \oplus X_i = [0 \ 1 \ 1 \ 1 \ 0 \ 0 \ 1] = V_1$$

$$P_{best}_i \oplus X_i = [1 \ 0 \ 0 \ 1 \ 0 \ 1 \ 0] = V_2$$

$$R_1 = r_1 c_1 = 1.4, \quad R_2 = r_2 c_2 = 0.7$$

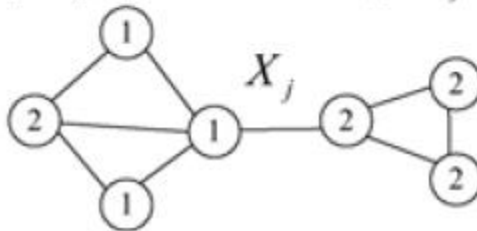
$$R_1 V_1 = [0 \ 1.4 \ 1.4 \ 1.4 \ 0 \ 0 \ 1.4] = V_3$$

$$R_2 V_2 = [0.7 \ 0 \ 0 \ 0.7 \ 0 \ 0.7 \ 0] = V_4$$

$$V_3 + V_4 = [0.7 \ 1.4 \ 1.4 \ 2.1 \ 0 \ 0.7 \ 1.4] = V_5$$

$$\text{sig}(V_5) = [0 \ 1 \ 1 \ 1 \ 0 \ 0 \ 1] = V_6$$

$$X_i \otimes V_6 = [1 \ 2 \ 2 \ 1 \ 2 \ 2 \ 2] = X_j$$





# 算法框架

## ➤ 粒子初始化

### PGLP算法

$$l(i) = \operatorname{argmax}_r \sum_{j \in \Omega(i)} \delta(l(j), r)$$

#### Algorithm 2 Pseudo Code of PGLP

```

Begin
Input: label propagation iterations: iters.
  for each chrom(i) ∈ population
    for(j = 1 : iters)
      for(k = 1 : vertexes)
        if(node[k].neighbor.size > 1)
          for(m = 1 : node[k].neighbor.size)
            Node[k].label ← formula (3);
          else Node[k].label ← node[k].neighbor.label;
        End for
      End for
    End for
  End for
Output: initialed chromosomes.
End

```

## ➤ Gbest

从  $n_s$  个 corresponding neighbors 随机选择





# 算法框架

## ➤ 契比雪夫方法 Tchebycheff approach

### MOEA/D

- ◆ Weighted Sum Approach
- ◆ Tchebycheff Approach
- ◆ Boundary Intersection (BI) Approach

$$g^{te}(x|\omega, z^*) = \max_{1 \leq i \leq k} \omega_i |f_i(x) - z_i^*|$$

$$x \in \Omega$$

$$z^* = (z_1^*, z_2^*, z_3^*, \dots, z_k^*)$$

$$z_i^* = \{\min f_i(x) | x \in \Omega\}$$





# 算法框架

## ➤ Pbest

If 新生成的解X 支配 pbest

Pbest = X;

else if pbest 支配 X

pbest保持其原始状态;

else //互相不支配

// 那么我们使用聚合方法来确定是否更新pbest

If  $w_{i1}f(x_i^{t+1}) + w_{i2}f(x_i^{t+1}) < w_{i1}f(pbest_i) + w_{i2}f(pbest_i)$

pbest = x;

end if

end if

end if





# 算法框架

## ➤ 震荡操作 turbulence operation

- ◆ 维持多样性
- ◆ 逃离局部最优

### 基于邻域的突变 (NBM)

对每个粒子中的节点，如果随机数小于突变概率 $pm$ ，则将NBM过程应用于该节点，即，将其标签标识符分配给其所有邻居。

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**Algorithm 3** Pseudo code of turbulence operation on one particle.

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```
1: for  $i = 0; i < vertex; i ++$ 
2:   if  $rand(0, 1) < pm$ 
3:     for  $j = 0; j < node[i].neighborsize; j ++$ 
4:        $x[node[i].neighbor[j]] = x[i];$ 
5:     end for
6:   end for
7: end for
```

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# 算法框架

## ➤ MODPOS算法整体框架

输入：网络的邻接矩阵A

输出：网络划分结果

Step1. 初始化

step 1.1 初始化种群  $P = \{x_1, x_2, \dots, x_{pop}\}^T$

step 1.2 初始化速度  $V = \{v_1, v_2, \dots, v_{pop}\}^T$

step 1.3 初始化分布均匀的权重向量

step 1.4 初始化  $pest, pbest_i = x_i$

step 1.5 初始化参考点  $z^*$

step 1.6 初始化neighborhood N 基于欧拉距离

$N = \{n_1, n_2, \dots, n_{pop}\}^T$

Step 2. 设置迭代次数标记  $t = 0$

Step 3. 循环开始 for  $i = [1, pop]$  do

step 3.1 选择  $gbest$

step 3.2 计算速度

step 3.3 计算位置

step 3.4 if (  $t < maxgen * pm$  ) 突变

step 3.5 计算新解  $x(i)^{t+1}$

step 3.6 更新neighborhood

step 3.7 更新参考点  $z^*$

step 3.8 更新  $pest$

step 4 if (  $t < maxgen$  ) {  $t++$ ; go to step3; }

else output





# 实验

## ► 规范化互信息 NMI

通过比较网络真实社区结构与MODPSO发现的网络社区结构的结构相似性来验证算法的有效性。

If  $NMI = 1$  , 两种社区结构完全一致

If  $NMI = 0$  , 两种社区结构完全不同

$$NMI = \frac{-2 \sum_{i=1}^{C_A} \sum_{j=1}^{C_B} C_{ij} \log(C_{ij} N / C_i \cdot C_j)}{\sum_{i=1}^{C_A} C_i \log(C_i / N) + \sum_{j=1}^{C_B} C_j \log(C_j / N)}$$





# 实验

## 实验结果

网络	节点数	边数	真实社区个数
dolphin	62	159	2
football	115	613	12

网络	指标	值
dolphin	NMI <sub>max</sub>	1
	NMI <sub>avg</sub>	1
	Q <sub>max</sub>	0.4198
	Q <sub>avg</sub>	0.4198
football	NMI <sub>max</sub>	0.9289
	NMI <sub>avg</sub>	0.9278
	Q <sub>max</sub>	0.6046
	Q <sub>avg</sub>	0.6035





# 总结与展望

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## ➤ 总结

- 对于离散问题重新定义粒子的速度和位置
- 基于分解的多目标网络聚类方法
- 将网络拓展至符号网络

## ➤ 展望

- 引入分布式并行计算





Thank you !

